1. {6 marks} Fix $k \geq 1$. Find the generating series for binary strings with no 01^k0 substring (where 1^k means k copies of 1). Justify your decomposition and write your generating series as a ratio of polynomials.

2. {6 marks} Consider the set C of binary strings which includes the empty string and for which every nonempty element w of C, the first bit of w is 0, the last bit of w is 1, and the rest of w consists of a concatenation of zero of more elements of C.

   Use the recursive decomposition technique to find an equation which the generating series of C satisfies. You do not need to solve your equation for $\Phi_C(x)$.

3. {5 marks} Solve the recurrence $a_n = -a_{n-1} + 2a_{n-2}$ for $n \geq 2$ with initial conditions $a_0 = 2$, $a_1 = 3$.

4. {5 marks} Solve the recurrence $b_n = -3b_{n-1} + 4b_{n-3}$ for $n \geq 3$ with initial conditions $b_0 = 9$, $b_1 = -9$, $b_2 = 18$.